SCI-BONO DISCOVERY CENTRE
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GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA
Kelello
UNIVERSITY OF JOHANNESBURG

FP training manual: Multiplicative reasoning

30th May 2017

Kelello, in partnership with Centre for Education Practice Research (CEPR), University of Johannesburg

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Acknowledgements

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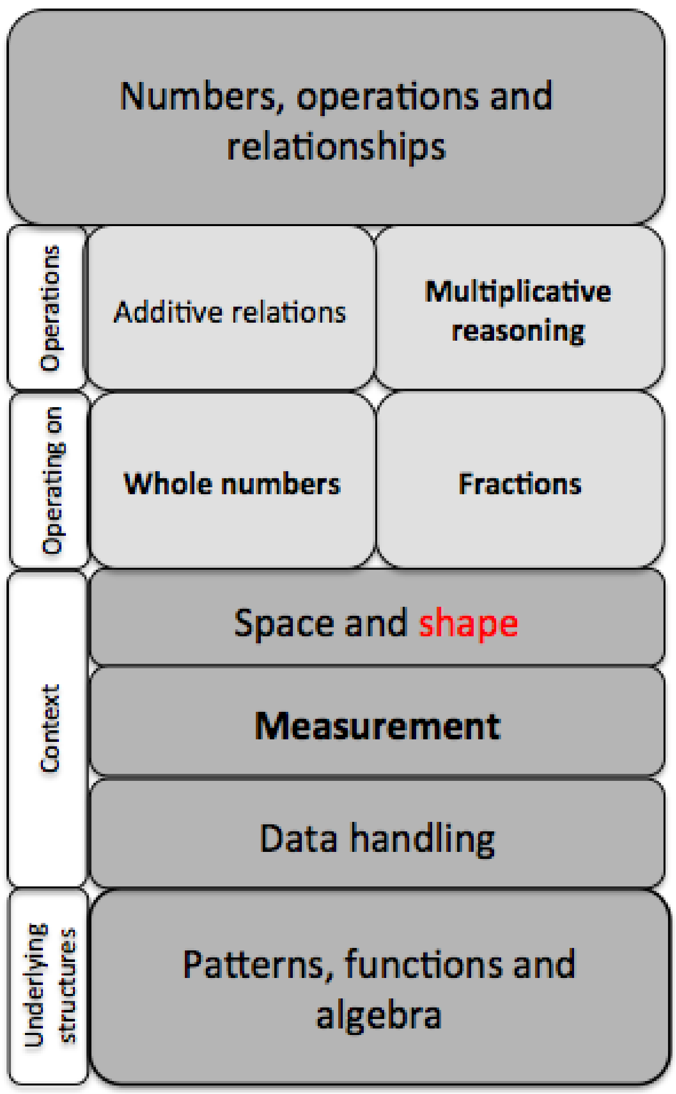
Contents

Introduction

Welcome to the Sci-Bono training workshop for lead teachers of Gauteng Department of Education Foundation Phase teachers. This manual is a guide for training teachers in how to teach multiplicative reasoning in the Foundation Phase.

Training objectives

By the end of the training, participants should have embarked on a professional pathway towards ongoing reflective practice in mathematics teaching. There is a specific focus on multiplicative reasoning (situations and calculations which require dividing or multiplying). However, this topic fit into a bigger picture of mathematics. None of the topics in mathematics can stand totally alone, and connection between topics and sub topics are very important.



During the 12-hour training workshop, participants will:

1. Solve problems about **multiplicative reasoning**, using contexts that encourage/highlight:
    - A number of different representations as tools for mathematical thinking and solving problems:
        - Number lines;
        - Whole equal part diagrams;
        - Arrays; and
        - T-tables (and clueboards).
    - The difference between multiplication and division, how division and multiplication relate (go together or are inverses);
    - The difference between grouping and sharing situations for division;
    - The difference between problems resulting in a remainder and those resulting in a fraction; and
    - Identify the unit of repetition;
2. Reflect on **pedagogical content knowledge** in terms of:
    - Likely ways in which children will represent (act-out; retell or draw) problem situations;
    - Allowing children to think, act and draw (and being flexible about how they do this) as children make sense of problems for themselves;
    - Inducting children into using powerful diagrams/images in mathematics which are useful for multiplication and division in the higher grades;
    - Distinguishing different types (archetypes) for multiplying and dividing situations and so how to pose your own problems for learners...

Workshop programme

SATURDAY 13 MAY 2017

07:30 – 8:00	Arrival tea/coffee
08:00 – 9:00	Admin Activities & ice breaker
09:00 – 09:30	Pre-test
09:30 – 10:00	**TEA/COFFEE**
10:00 – 12:30	TASKS 1: Why are representations important? TASKS 2-6: Number lines and whole equal part diagrams
12:30 – 13:15	**LUNCH**
13:15 – 15:15	TASKS 7-10: Posing and solving problems TASKS 11-15: Grouping and Sharing
15: 15 –15: 45	**TEA/COFFEE**
15: 45 – 17:15	TASKS 16-17: Classification TASKS 18-20: Arrays

SUNDAY 14 MAY 2017

07:30 – 08:00	Arrival Tea/Coffee
08:00 – 10:15	TASK 21-23: Arrays continued TASKS 24-26: T-tables
10:15 – 10:45	**TEA/COFFEE**
10:45 – 12:15	Consolidation and revision
12:15 – 12:30	Evaluation
12:30 – 13:00	Post-test

Mental maths

An important part of mathematics at all levels is to practise fluency. One way to facilitate this is to start every day with some fun mental maths activities. Mental maths should get children talking, thinking and acting using mathematics. It 'switches on' their brains for thinking mathematically. During the training, we will start each session with a mental maths activity.

CAPS explains the role that mental mathematics plays in the curriculum:

> *Mental mathematics plays a very important role in the curriculum. The number bonds and multiplication table facts that learners are expected to know or recall fairly quickly are listed for each grade. In addition, mental mathematics is used extensively to explore the higher number ranges through skip counting and by doing activities such as "up and down the number ladder". These activities help learners to construct a mental number line.*

The mental maths activities in this manual will focus on skip counting. Once learners are comfortable with counting in ones they can start to learn how to skip count (for example, counting in 2s or in 5s). This all takes time and lots of experience. Over the four years of Foundation Phase, children should:

1. Learn to emphasise multiples (while still naming all numbers in ones).
   E.g. Emphasising the evens (counting pairs of eyes or pairs of feet): 1, **2**, 3, **4**, 5, **6**, 7, **8**, 9, **10**
2. They then learn to name only the multiples.
   E.g. Counting in twos (pairs of eyes or pairs of feet: 2, 4, 6, 8, 10)
3. When this is fluent, they can start to use skip counting to do calculations
4. And finally leads to efficient multiplication and division.

Depending on the numbers this happens at different grades. Skip counting in 2s and 5s and 10s can become fluent early (Grade R and Grade 1). Children who have experience of problems using groups of 2s, 5s and 10s can start calculating with these groups. Skip counting in 4s, or 3s needs more experience and takes time (Grades 2 and Grade 3). And skip counting in 6s, 7s, 8s and 9s even later (Intermediate Phase).

Mental Maths 1: Introducing skip counting

First learners name all the numbers but emphasise the multiples.

Counting in 2s

- Hit the table when you say the odd numbers and clap your hands when you say the even numbers.
- Bend down for odd numbers and jump up for even numbers.
- Say the odd numbers softly and the even numbers loudly.

- Take it in turns for the teacher and students to count, so they only say every other number in the sequence.
- Split the class in two – each half takes it in turns to say the next number.

Variation

- **3s**: hit table twice and clap hands once.
- **3s**: touch toes for 1, hips for 2 and jump up for multiples of 3, continue pattern.
- **5s**: touch toes, knees, hips, shoulder, head and then jump up for 5.
- **4s**: first count in 2s using the soft, loud, soft, loud technique to emphasise how counting in 4s is linked to counting in 2s.

Mental Maths 2: Skip counting

Children first learn to count in ones as a song or string or meaningless words. It takes time for them to separate the words and match each number word with a new object (a finger, concrete object or picture).

The same thing happens with counting in 2s, 5s, 10s or other multiples. It takes a lot of time and experience working with and arranging groups to connect the number names of multiples with groups (keeping track on fingers, touching or moving a repeated group).

Once learners are comfortable with saying only the multiples loudly and all other numbers softly, they are ready to practise saying the sequence without having to count "in their head". At this stage, they are skip counting. Below are a number of ideas for practising skip counting that move beyond chanting as a whole class and are aimed at keeping learners engaged.

Young children (Grade R and before) can learn to measure (repeating a unit) and reason about a situation involving multiplying or dividing situations (such as doubling or halving or sharing equally or packing into groups). If they don't yet know the skip counts, and how to keep track (associate each count with a beat/count) then they work out problems by arranging, drawing and acting on ones. They need to be able to talk, act, touch, arrange and draw using repeated groups to make sense of problem situations.

Taking Turns

- Divide the class into groups (e.g. each row becomes a group) and take turns to say the next multiple.
- Instead of saying the multiples in order (first row says first multiple, second row says second multiple etc), the teacher points to the group (or individual) that must say the next multiple.

- **5s**: split the class in two – one group says 5, the next group 10, the first group says 15, the second group says 20, continue pattern. This emphasises that when counting in 5s there is a simple pattern that repeats
- **10s**: point to the 100 chart while counting

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Discussion guide

When children are comfortable with skip counting (in 2s or in 5s say). They should practise skip counting starting at 0. This is important for calculation strategies that involve repeated addition on the number line (see Task 2).

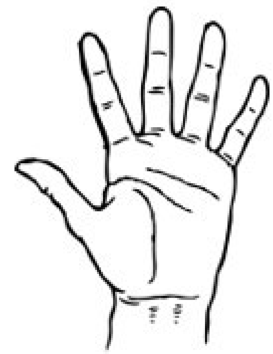
Mental Maths 3: Skip counting with fractions

Skip counting doesn't only have to be with whole numbers, for example, measurement contexts can be used to help learners count in halves. This appropriate for Grade 2 or Grade 3.

Mental Maths 4: Skip counting to calculate

Once learners are comfortable skip counting, the skip counting can be used to do calculations such as those suggested below:

How many?



There are 5 learners in the group. How many eyes are there?

There are 6 chairs, how many legs are there?

There are 5 learners in the group. How many fingers are there?

Variations

There are 24 eyes. How many learners are there?

There are 20 chair legs, how many chairs are there?

There are 35 fingers pots, how many hands are there?

Discussion guide

Think of other things that come in 2s, 3s, 4s, 5s and 10s that you can use to practise skip counting as a way of calculating.

Mental Maths 5: Games

Games are another way of practising fluency. These two games are to practise multiplying and dividing. The games show that multiplication and division are inverses of each other. To undo multiplication, you have to divide. To undo division, you have to multiply.

The following game brings together a number of ideas: taking turns while skip counting, skip counting that doesn't start at 0, allowing learners to choose number to start at and number to stop at.

Hands up

Work with a small group (about 10 learners) or divide the class into groups (e.g. each row becomes a group).

Choose a number to count in (e.g. counting in 10s).

Choose a multiple of that number to start at (e.g. 30).

Choose a multiple of that number to finish at (e.g. 120).

The first person says at the 'start' multiple (30). The second person says the next multiple (40). Once every person (or group) has had a turn, the first group says the next multiple and so on.

The person (or group) who says the 'end' multiple (120) puts up one hand.

The next person in the sequence says the 'start' multiple again (30) and the skip counting continues.

The first person to put up both hands is the winner.

Variations

- Count backwards
- Start (and end) on a number that is NOT a multiple of the number you are skip counting in e.g. count in 10s but start at 1. (Grade 2/3):

1, 11, 21, 31, 41, 51, 61, 71, 81, 91, 101, …

I'm thinking of a number…

- When I double my number I get 6. What is my number?
- When I halve my number I get 4. What is my number?
- When I repeat my number 3 times and add together I get 12. What is my number?
- When I triple my number 3 I get 12. What is my number?

What is my secret?

The teacher decides on a rule (e.g. double) and whispers the rule to one learner.

The class has to guess the rule.

To do this they say a number (e.g. 7) and the learner says the number she gets by applying the rule (e.g. 14).

This continues until the class works out the correct rule.

Rules that use multiplication reasoning that are suitable for Foundation Phase:

- double
- triple
- halve
- times by 10
- times by 5

Discussion guide

Multiplicative reasoning means all mathematics that is about multiplying or dividing. There is a unit that repeats. The unit can be a group or a unit of measurement. It includes

- Doubling and halving;
- Thinking about multiplication or division situations (sharing equally or packing into equal groups, arranging in equal rows);
- Solving and posing multiplication or division problems;
- All measurement contexts where you use
  - an informal unit (eg a hand or foot or piece of string or youghart cup) or
  - a formal measurement (metre or litre or cup);
- Fractions – where there is a whole and a equal parts, or where the the unit that is repeated is a fraction (eg half a cup,or half a metre).
- Ratio where there is a unit repeated. For example 2, suckers cost R6. How much will 10 suckers, or 3 suckers cost?

Mathematical thinking

We all have slightly different ways of thinking expressing and imagining mathematics. Luckily, we learn mathematics in groups and so can share our ways of thinking with each other. By seeing how others approach something we can often improve our own methods.

Children think differently to adults and are not yet using clearly defined objects, images or concepts that adults have already accepted. So, they need time to imagine and express themselves. This involves talking, acting, drawing, writing, arranging things. Sharing what has been done and explaining to another child (and perhaps an adult) is very important part of their development.

Task 1: Drawings

We often use mental drawings and or remember situations which we adapt, when we are thinking about something new.

What do you imagine for: ‘Five times four’ or ‘five multiplied by four’?

You could think of a situation, a picture, number symbols, a way of working this out.

THINK: Be still. Give yourself time to think. Think in your other languages.

MARK: On your own, write down or draw some of your ideas on scrap paper

REHEARSE: Practise what you will say to a partner about your drawings/markings/ writings.

RECORD: Make a neater version of your work to share with the group.

Discussion guide

Mathematics is about getting time to think. When we have our thoughts, we can then share them. We share with ourselves. Then we share with others. All humans have the power to imagine and express.

As mathematicians, we then communicate our ideas. We try and convince others of our ideas. All humans have the power to talk and to draw and act. We do this to communicate our ideas.

Often as teacher we do not give children time to imagine and express. We don't give them time to talk, draw and act. Children must communicate with other children. They must communicate with us. This is what they need to become mathematicians.

Following these steps can slow maths down:

1. THINK (in many languages),
2. MARK (a private time to express to self)
3. REHEARSE (practicing in one or many languages, what to say to another)
4. RECORD (making notes or makings to communicate to someone else).

Slowing down gives time to think mathematically.

In the marking and recording stages we often use drawings or writing. These are powerful human tools. They can be unique to you. They can be something shared over generations (eg writing text or using number symbols).

As a teacher, we need to give children time to invent their own markings. Children should experiment and try for themselves. But then we also need to help them learn about our culture. We have ways of writing and drawing which has been passed down through generations and across cultures. These are representations – ways of showing an idea or thought – in a recorded (written or drawn) form. Our job as teachers is to help children come to know the powerful representations for mathematical ideas.

Number lines as representation

We can choose a problem situation to introduce a powerful representation.

Task 2: Frog family

Mommy frog and baby frog go for a walk. When mommy frog does one big jump, baby frog does two small hops to get to the same place.

a) If mommy frog does 5 jumps, how many hops does baby frog have to do?

Daddy frog takes big jumps. When he takes one big jump, baby frog takes five little hops to get to the same place.

b) If daddy frog does 3 jumps, how many hops does baby frog have to do to get to the same place?

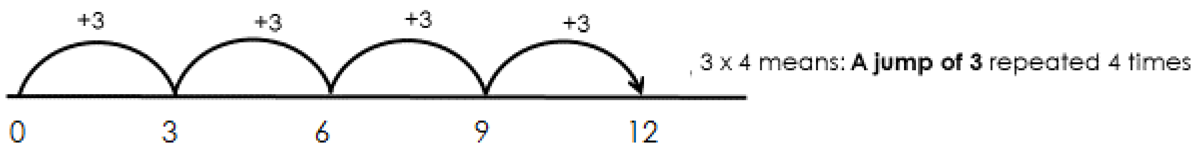
c) What other stories can you tell that involve small hops and big jumps?

Discussion guide

The number line is a useful tool (representation) for solving problems that require repeated addition (or repeated subtraction – see Task 3).

It is useful to distinguish between hopping in ones and jumping in units that are bigger than one. For example, the mother frog does one **jump** for every two **hops** that the baby frog does.

Repeated addition is one way of making sense of multiplication. With multiplying, you repeat equal jumps. The jumps stay the same size and you move to the right: for example, multiplying by 3 is imagined as jumps of 3 repeating to the right, and 3 × 4 is pictured as '3 repeated 4 times'. You start at zero and make jumps of 3: you land on 3, 6, 9, and 12.



A common misconception (if multiplication is done on a number line without a context) is to start at the first multiple rather than starting at 0. This results in the answer (the number of hops) being one less than the correct answer. Because of this it is important, once learners are comfortable skip counting from the first multiple, to practise skip counting from 0.

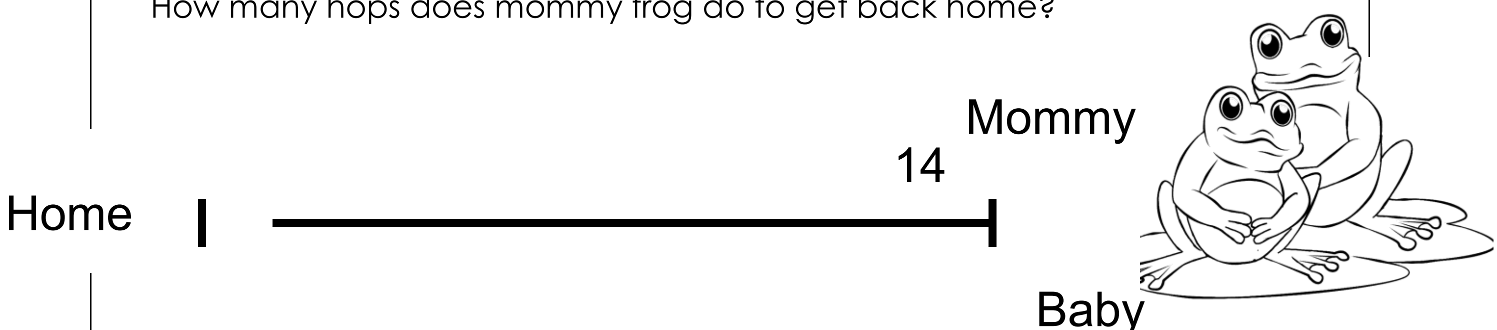
Other contexts that lend themselves to the number line as representation include rabbits hopping, kangaroos jumping, children walking.

Task 3: Go back home

A: Baby frog and mommy frog go for a walk.

When mommy frog does one hop, baby frog does two hops to keep up.

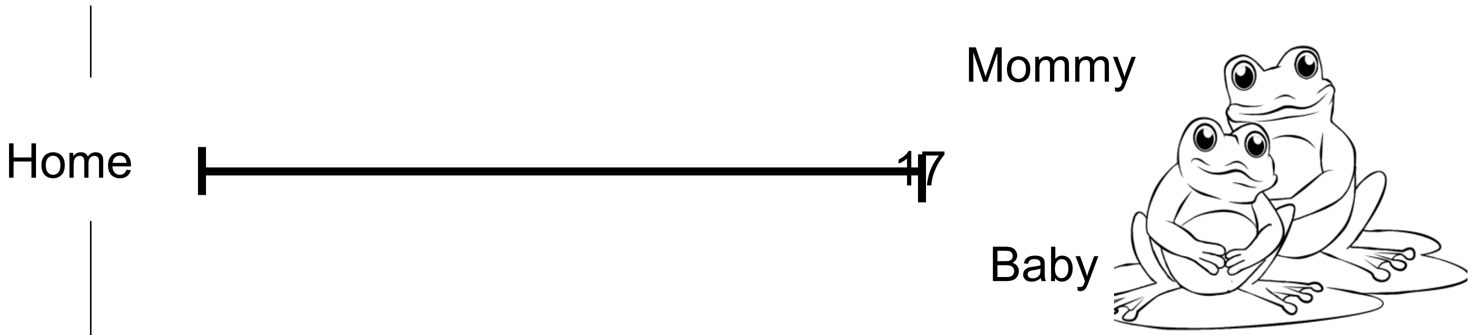
After baby frog hopped 14 times they hear daddy frog calling for help. How many hops does mommy frog do to get back home?



B: Baby frog and mommy frog go for a walk.

When mommy frog does one hop, baby frog does two hops to keep up.

After baby frog hopped 17 times they hear daddy frog calling for help. How many hops does mommy frog do to get back home?

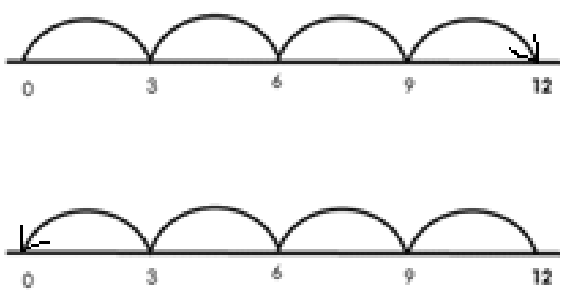


What is the same and what is different between **A** and **B**?

Discussion guide

These two examples illustrate how a number line can be used to solve a problem that requires repeated subtraction. Repeated subtraction is one way of thinking about division.

With dividing, you repeat equal jumps too – but now you jump backwards. For example, 12 ÷ 3 can be pictured as the number of jumps of 3 backwards from 12 to reach 0. It can also be pictured as the number of jumps of 3 from 0 to reach 12.



12 ÷ 3 can mean:

(a) how many jumps of 3 are repeated to reach 12? (i.e. start at 0, and jump until 12)

OR

(b) how many jumps of 3 back from 12 are needed to reach 0? (i.e. start at 12 and jump backwards to 0)

Once learners are comfortable using a number line, it is also possible to use the equal jumps approach to understand remainders. For example, to find 14 ÷ 3 one can make 4 jumps of 3 to reach 12. Another jump will take you too far, so that is not possible. So, the difference between 12 and 14 is the remainder (i.e. 14 ÷ 3 = 4 rem 2).

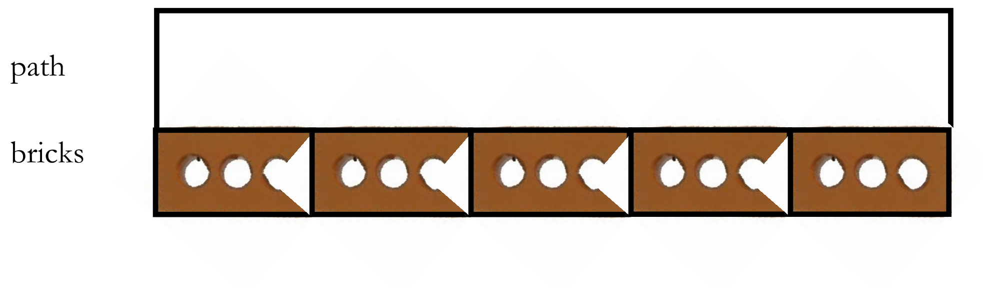
The remainder is the difference between the two problems: **B** results in a remainder and **A** does not. We see that the remainder for B is 1 because mommy frog is 1 away from 0 when she has done 8 jumps. She can't take another jump without going past their home.

Whole-equal part diagrams

Task 4: The Bricklayer

Nosiphiwo is a bricklayer and has a path that she wants to line with bricks.

She uses 5 bricks to line the path. Each brick is 3 units long. How many units long is the path?



Discussion guide

This context introduces the representation of a 'whole-equal part' diagram which is another way of representing multiplication and division.

whole		
part	part	part

6		
2	2	

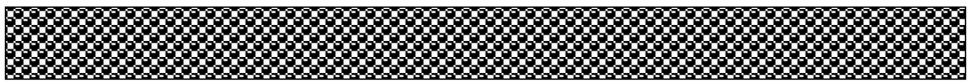
It is important to notice that the parts are all **equal**.

This representation emphasises that multiplication can be see was repeated addition.

3 x 2 = 2 + 2 + 2 = 6.

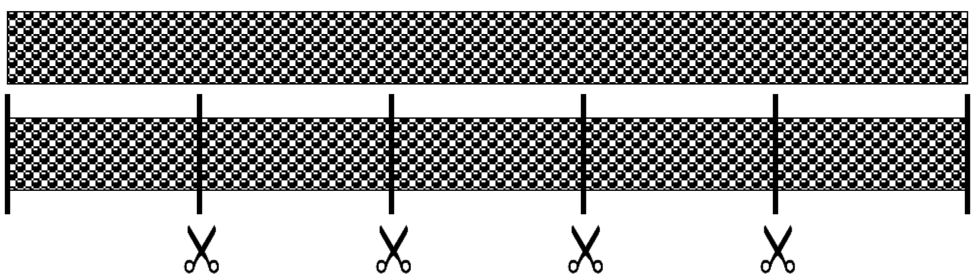
Task 5: The dressmaker

Thabo is a dressmaker and has 10m of material. He needs 2 metres to make one dress. How many dresses can he make?



Discussion guide

This is another context that lends itself to using a whole-equal part diagram. However, this time the whole is being divided into equal parts rather than the whole being constructed by putting together equal parts (as was the case in Task 4).



Whole-equal part diagrams can be used to visualise the relationship between multiplying/dividing. This image can also help to make the connection between fractions and division very explicit.

whole		
part	part	part

whole = 3 x part   part = whole ÷ 3

For example:

Whole-equal parts diagram

12		
4	4	4

Family of number sentences

4 x 3 = 12   3 x 4 = 12

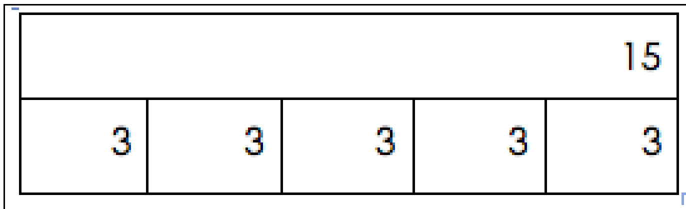
12 = 4 x 3   12 = 3 x 4

$12 \div 3 = 4$ $12 \div 4 = 3$

$4 = 12 \div 3$ $3 = 12 \div 4$

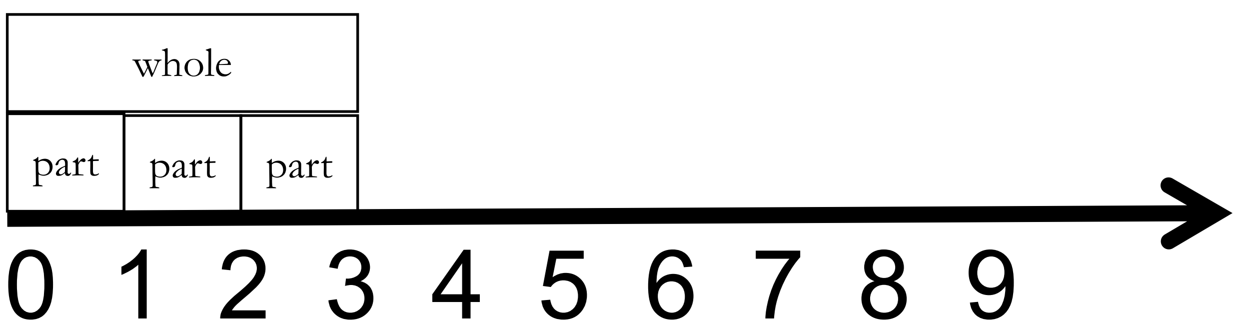
Task 6: Multiplication and division

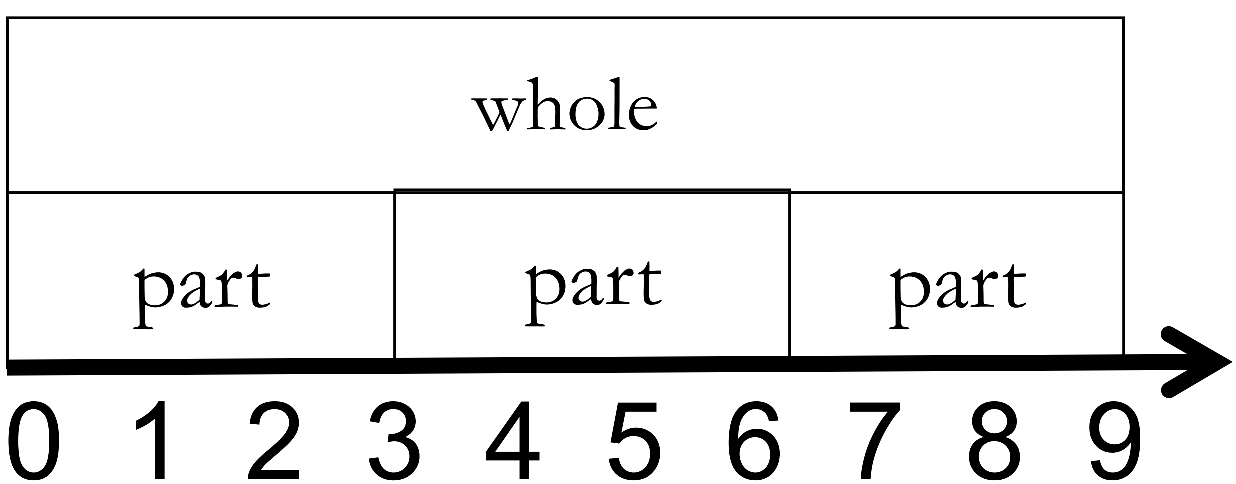
Write down 8 equations involving multiplication/division that can be created from the whole-equal parts diagram below.



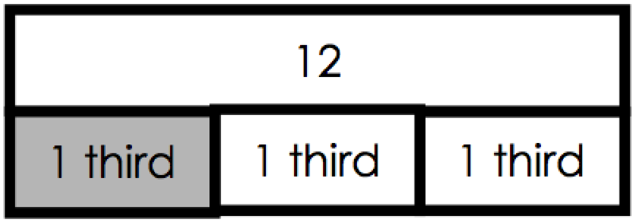
Discussion guide

It is also possible to see the whole-equal part diagram in conjunction with the number line. The two examples below show how 3 divided into 3 equal parts is 1 and how 9 divided into 3 equal parts is 3.





Alternatively, division can be thought of as finding the fraction of a whole:



1 third of 12 means 12 is split into 3 equal parts

Make 3 equal jumps to reach 12.
How big is each jump?

Posing and solving problems

Every story has a location, characters, props, something that happens (an action) and a challenge that needs to be overcome. There is a problem that needs to be solved. It is useful to think of these five characteristics of a story when posing word problems. And to be aware of them when solving word problems.

Task 7: Who, what, where, why?

A: Nosiphiwo is a bricklayer and has a path that she wants to line with bricks. She uses 5 bricks to line the path. Each brick is 3 units long. How many units long is the path?

B: Daddy frog takes big jumps. When he takes one big jump, baby frog takes five little hops to get to the same place. If daddy frog does 3 jumps, how many hops does baby frog have to do to get to the same place?

a) Identify location, characters, props, action and challenge in **A** and **B**.

b) Which of the five characteristics can be changed without changing the calculation that needs to be done to solve the problem?

Discussion guide

Make sure that all the characters are identified (the daddy frog and the baby frog) and that all the props are identified (the bricks and the path). Sometimes the props are not physical things like in **B** where they are the jumps and the hops.

Changing some characteristics of a word problem has a different effect to changing other characteristics. For example, if we change the **location**, the **characters** or the **props** the calculation that needs to be done to solve the problem doesn't change. If we change the **action** the calculation might or might not change.

Task 8: Are they the same story

A: Thabo is a dressmaker and has 15m of material. He wants to share the material equally between his five daughters. How long will each of the pieces be?

B: Mirriam is a baker and has 15 rolls. She sells 5 of them. How many rolls does she have left?

C: Thabo is a dressmaker and has 15m of material. He sells 5m. How much material does he have left?

D: Mirriam is a baker and has 15 rolls. She wants to share the rolls equally between her five sons.

a) Identify which pairs of stories result in the same calculations.

b) Pose two word problems that have different locations, different characters and different props but which result in the same calculation.

Discussion guide

A and D both result in the calculation 15 ÷ 5.

B and C both result in the calculation 15 – 5.

Task 9: Changing the action

Thami has 5 apples. He eats 3 of them. How many apples does he have left?

a) What is the action in the word problem?

b) Think of *different* actions for the word problem that result in the *same* calculation.

c) Think of *different* actions for the word problem that result in a different calculation.

d) Pose a question with a different action that results in a different calculation.

Discussion guide

In this word problem, the action is 'eating' and this results in the calculation 5 – 3. Other actions resulting in the same calculation can include 'loosing', 'giving away', 'being stolen' etc.

Examples of actions that result in a different calculation are 'finding more', 'being given more', 'buying more'. For example: *Thami has 5 apples. He finds 3 more apples. How many apples does he have now? [action: finding]*.

Note that the phrasing of the problem changes from 'How many apples does he have left?' to 'How many apples does he have now?'

One variable that we have not discussed is the NUMBER of props.

Task 10: Changing the numbers

A: Thabo is a dressmaker and has 15m of material. He wants to share the material equally between his five daughters. How long will each of the pieces be?

B: Thabo is a dressmaker and has 15m of material. He wants to share the material equally between his three daughters. How long will each of the pieces be?

a) What is the same and what is different between the two word problems?

b) How is the calculation that needs to be done in **A** and **B** different and how is it the same?

Discussion guide

Although the bare calculation is different, the type of calculation is the same. These two problems are therefore considered the same 'type' of problem. In the next section we will look at different types of problems that have to do with multiplicative reasoning.

Sharing and grouping

In this section, we focus on the difference between grouping and sharing. We will use several investigations which you can work through, and then use with your class.

Task 11: There's a fire on the mountain

11 learners run around a table (or around a number of chairs or rocks if playing outside). Teacher shouts: Make groups of 2. Learners have to make groups of 2. If a learner isn't part of a group they are out. Teacher asks: How many groups of 2 did we make?

Play again with 11 learners but make groups of 5. How many groups of 5? How many learners are not in a group?

Discussion guide

This game allows learners to experience the making of groups. It also introduces the language needed for groups: __ groups of __. Sometimes making groups results in a remainder, sometimes it doesn't.

Task 12: Marshmallow sandwiches

Sive is making marshmallow sandwiches. For each sandwich, he needs two marshmallows.

A: Sive has 10 marshmallows. How many marshmallow sandwiches can he make?

B: Sive has 11 marshmallows. How many marshmallow sandwiches can he make?

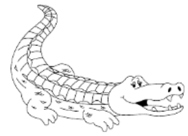
a) Solve each of the problems.

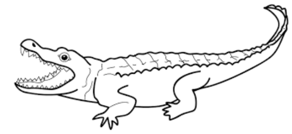
b) What is the same and what is different between task A and B?

Discussion guide

This is another example of a **grouping** problem. Instead of grouping learners, marshmallows are being grouped. As with the learners, sometimes there is a remainder and sometimes there isn't.

Task 13: Crocodile picnic





C: Two crocodiles go on a picnic. They have 10 marshmallows. How many marshmallows will each crocodile get if they share them equally?

D: Two crocodiles go on a picnic. They have 11 marshmallows. How many marshmallows will each crocodile get?

E: Two crocodiles go on a picnic. They have 11 suckers. How many suckers will each crocodile get?

a) Solve each of the problems.

b) What is the same and what is different between task C and D?

c) What is the same and what is different between task D and E?

Discussion guide

The problems in this task are **sharing** problems. They are different to the problems in Task 11 and Task 12 which are grouping problems. Like with the grouping problems, some problems result in remainders and some don't. In the sharing problems, it is easy to see that sometimes the remainder can be divided so that a fraction is created (as with the marshmallows) and other times the remainder can't be divided (as with the suckers).

Task 14: Grouping vs Sharing

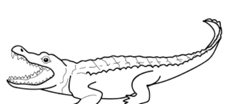
a) What is the same and what is different between task A and C?

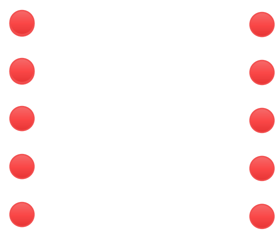
b) What is the same and what is different between task B and D?

Discussion guide

It is important, as a teacher, to be aware that grouping and sharing result in the same answer. Sharing 10 things between 2 people (each person gets 5) gives the same result as having 10 things and making groups of 2 (each group has 5 things).

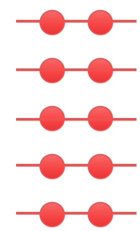
The grouping and sharing examples we have covered use groups of **two** or share things between **two** people (or two crocodiles). Sharing between two is the same as halving. Halving and doubling are an important part of Grade 1 and learners should have many opportunities to double and to halve. The advantage of halving is that it is possible to use a mirror line to indicate the halving.





2 groups of 5

Each crocodile gets 5 marshmallows



5 groups of 2

There are 5 marshmallow sandwiches

By the end of grade 3 learners should appreciate that sharing and grouping are equivalent. Learners should start by sharing out marshmallows one at a time, but should be supported to realise that they can count out 2 at a time (one for each crocodile) which is the same as making groups of 2.

Task 15: Mangos in boxes

Thami has a factory packing mangoes. Two mangoes in a box. He has 11 mangoes.

a) How many boxes does he need?

b) How full is the last box?

c) How does this problem differ from problem **D** in Task 11?

Discussion guide

We have seen a **sharing** problem that results in a fraction as an answer (**D** in Task 13).

Task 15 is a **grouping** problem that results in fraction because of the context and because of the question asked (how full is the last box). The answer to the question 'how full is the last box' will be a fraction depending on how many mangos have been put in a box that has space for 3 mangoes. It is also important to note that b) requires the answer to be rounded up as a whole box is required even if the box isn't full.

Classification of problem types

Task 16: Types of problems

Read each division problem (see table below with 6 questions), and then:

a) Write down for each one whether it is a **sharing** or a **grouping** problem.
b) Solve the problems. Show your work.

A: A teacher gave out 24 pens to 8 children in her class. Each child got the same number of pens. How many pens did each child get?	**B:** A baker made 30 cookies and placed them in boxes; 3 in each box. How many boxes of cookies did he fill?
C: A gardener had 28 seeds to plant in pots, and planted 7 in each. How many pots did she plant?	**D:** A class of 64 children was divided into relay teams. There were 4 children in each team. How many relay teams were created?
E: A shopkeeper was arranging 24 books for a display, and placed them in 6 rows with equal numbers of books in each. How many books were in each row?	**F:** A farmer had 27 kg of food for his 9 goats. Each goat was given the same amount of food. How much food did each goat get?

Discussion guide

Distinguishing between grouping and sharing problems is the first level of classification. None of the examples in this task result in a remainder. However, we have seen that both sharing and grouping problems can result in remainders. And that sometimes the remainder can be shared (resulting in a fraction) and sometimes it can't. This second level of classification is addressed in Task 17. Answers: Sharing: A, C, F and Grouping: B, D, E

Task 17: Types of problems

Below are the 6 different types of sharing and grouping word problems.

Sharing	**Grouping**
No remainders I have 12 mangoes. I share them equally between 3 people. How many mangoes does each person get?	**No remainders** I have 12 mangoes. I pack them into bags. There are 3 mangoes in each bag. How many bags do I need?
With remainders I have 13 balloons. I share them into 3 equal bunches. How many balloons in each bunch?	**With remainders** I have 13 mangoes. I pack them into bags with 3 in each bag. How many bags do I need to carry all the mangoes?
With fractions I have 13 cupcakes. I share them equally onto 3 plates. How many cupcakes on each plate? Can 3 people share the last cupcake? How much will each person get?	**With fractions** I have 13 mangoes. I pack them into boxes. There is space for 3 mangoes in each box. How many boxes do I need to carry all the mangoes? How full is the last box?

a) Make up your own example for each of the 6 different types of sharing and grouping problems. Keep the numbers the same.

b) Compare your 6 examples with a partner. Do you agree that their problems are examples of each of the 6 different types?

Discussion guide

It is important that teachers are aware of the different types of multiplicative reasoning problems in order to ensure that learners are given the opportunity to solve all the different types of problems in a meaningful context.

Children need time to make up their own problems too. Questions to help this include:

- Can you make a problem like the one we just worked on?
- Please change the numbers in this problem, but keep the situation the same.
- Please change the situation, but keep the numbers the same.
- Can you make an easier problem, like this one?
- Can you make a harder problem, like this one?

Array as representation

This section provides problems that introduce the array as a representation for multiplication and division.

Task 18: Cupcakes

Lewan is a baker, he bakes cupcakes. These are set out on trays. On a tray there are five cupcakes in a row. And there are eight rows of cupcakes.

How many cupcakes are there on a tray?

Discussion guide

Select learners to share their solutions with the class. Select those who have used the array clearly. Leave their solutions on the board (if necessary draw their solutions on the board for them – both for time saving and for accurate arrays)

What other array contexts could we use? Arrays are only formalised in Grade 3 but word problems that give rise to an array structure can be introduced earlier.

Powerful representations are the ways of recording thinking using writing or drawings to communicate an idea. Powerful representations are used in higher grades in school.

Children need time to imagine and express their own thoughts. Children must write and draw for themselves. Children also need time to come to understand and use the representations of others. Other people's representations are their friends/peers. The teacher also makes representations.

In a Foundation Phase in a school it is very helpful if all the teachers agree on what the powerful representations are that they will use in mathematics. When children first see a representation (eg an array) in Grade 1. The Grade 2 teacher also uses an array. Over time the children come to arrange objects in arrays. Eventually the representation is something that is thought (imagined) and not drawn.

With each new concept children should be encouraged to think and mark for themselves. Every maths lesson should have some discussion for learner to explain. They should share and compare ways of marking. We expect to see different representations in the children's work. They should not all be doing the same drawings.

Task 19: More cupcakes

Lewan makes cupcakes in five flavours – chocolate, plain, cherry, seed and lemon. He puts these in a row.

a) If there are eight cupcakes of each flavour, how many cupcakes are there altogether?

b) How is this problem different to task 18?

c) How is it the same?

Discussion guide

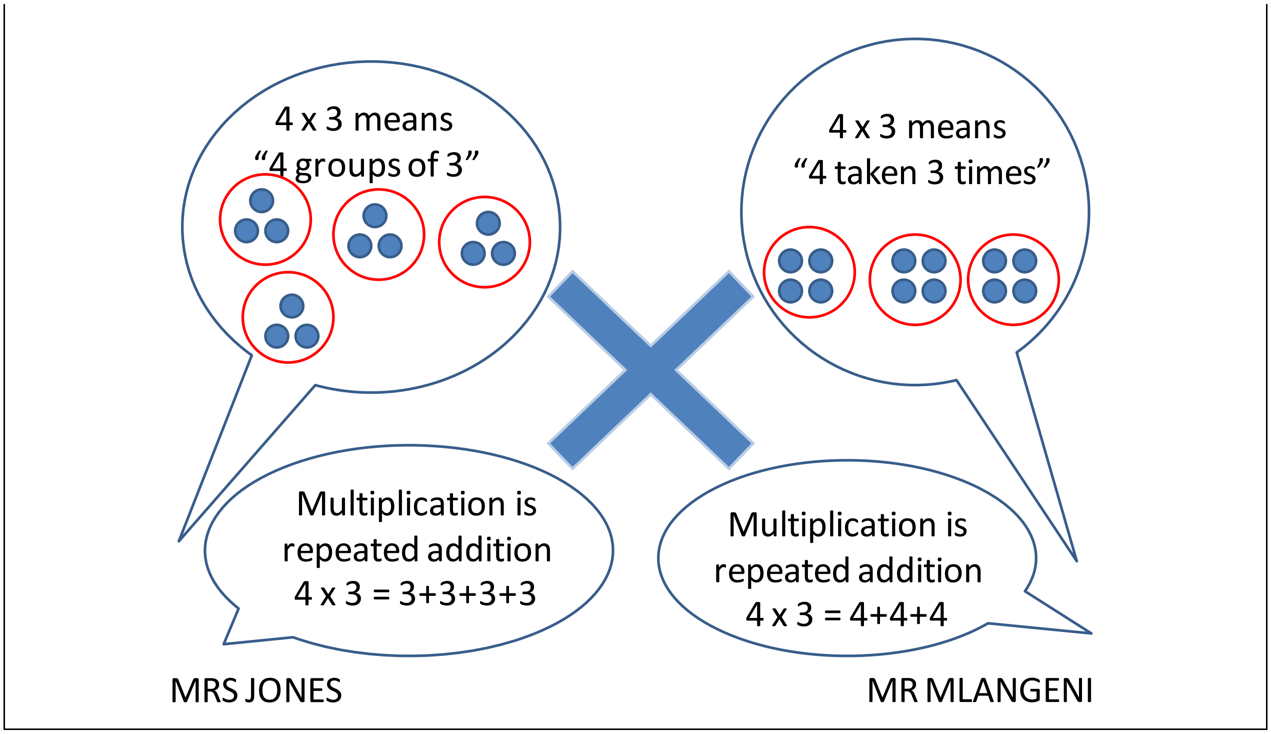
Discuss what is the same and different between the last two tasks. Notice that the numbers used and the solution is the same in both. But the problem situation is different. The action – what is done or acted out – is different.

Task 20: What does 3 x 4 mean?

1. Circle which are correct. 4 x 3 means:

a) 4 threes	b) 4 three times	c) 4 times three
d) 4 groups of 3	e) 3 lots of 4	f) 3 by 4
	g) 4 taken in 3 groups	

2. Two teachers explain what multiplication is. Who is correct, Mrs Jones or Mr Mlangeni?



Discussion guide

There is much debate about the difference between 3 x 4 and 4 x 3. Some people discuss whether learners should be taught a convention regarding how to think about ‘x’.

In England, many adults were taught 4 x 3 is ‘4 lots of 3’ [3+3+3+3].

In South Africa, many adults were taught ‘4 x 3 is ‘4 repeated 3 times’ [4+4+4].

How were you taught? How do you say ‘4 x 3’ in your language? Do you just say the numbers and symbol ‘4 x 3’ is ‘four times three’? Do you have another way you read/ say ‘4 x 3’?

The most important concept is that both ways make sense. Children – and teachers – need to know both ways. Both ways are equivalent.

Task 21: Dots

Put on the board the sheet showing 4 x 5 …(4 across and 5 down)

Learners agree in pairs how many dots they can see.

Agree that there are 20 dots.

Elicit, or introduce at least three methods of finding the total:

a. Count all the dots
b. Count four groups of five
c. Count five groups of four.

For the counting in groups methods, circle these groups on the sheet and record, for example, 4 groups of 5 = 20. If any learner uses the term 'times' record that as well. *(Leave these sheets on the board.)*

Repeat this for the sheet showing 5 x 4… (5 across and 4 down)

Talk about what is the same and what is different for 4 x 5 and 5 x 4.

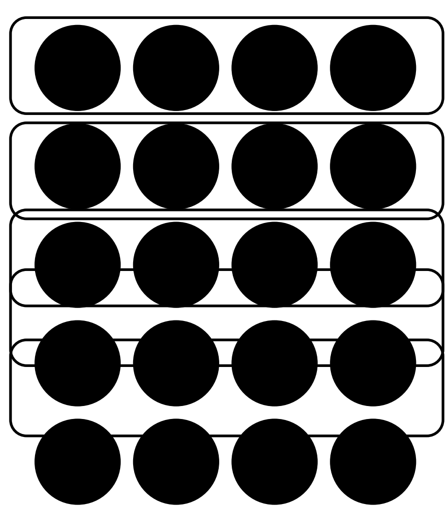
Discussion guide

An array (dots) helps to see that 5 x 4 is the same as 4 x 5. They are equivalent. Multiplication is commutative.

It can help to agree a convention on how to name an array: '5 x 4' is 5 across and 4 down. First say how many across, then how many down. Horizontal then vertical.

Show how to circle rows on the board with number of rows on one side and number of dots on the other side.

4 x 5, 4 across and 5 down



Number of rows	Number of dots
1	4
2	8
3	12
4	16
5	20

Repeat the same for 5 x 4, 5 across and 4 down

Record 1 Row is 5, 2 rows is 10, 3 rows is 15 etc.

Task 22: Even more cupcakes

Lewan is a baker, he bakes cupcakes. These are set out on trays. On a large tray there are five cupcakes in a row. There are 75 cupcakes altogether.

a) How many cupcakes are there in each column?

Lewan makes cupcakes in five flavours – chocolate, plain, cherry, seed and lemon. He makes the same number of each flavour.

b) If there are 75 cupcakes altogether, how many cupcakes are there of each flavour?

c) What is the same and what is different between the two stories/problems?

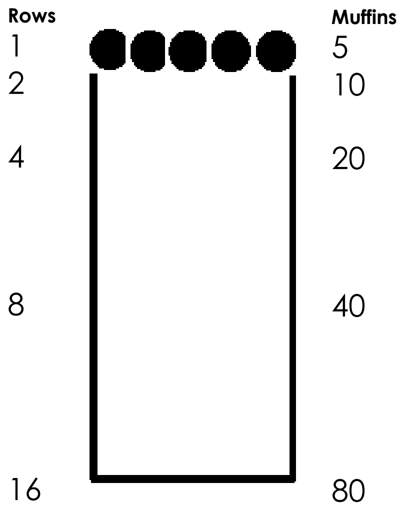
Discussion guide

What is the same and what is different between the two problem situations?

What is the same and what is different about the two array representations?

This problem can be used to show chunking. We don't have to only work one row or one group at a time.

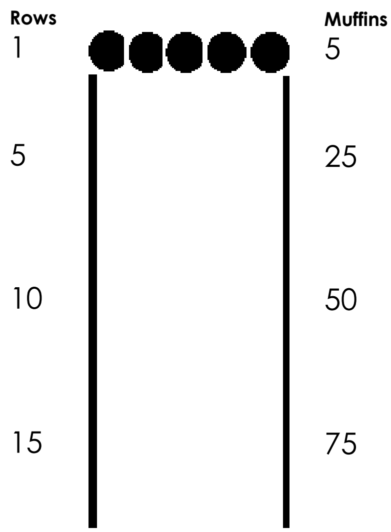
We need to reach 75. We could count in 5s using 1 group at a time: 5, 10, 15, 20, 25… but that will take a long time.



This uses doubling chunks.

You know double 5 is 10.

Then you double each time.



This uses chunks of 5 rows.

You know 5 fives are 25.

Then you repeat that chunk of 5 rows and 25 muffins.

And now we will apply arrays to a problem context.

Task 23: Coke bottles

Lihle collects empty bottles and keeps them in '6 by 4' crates such as the one in the picture.

Lihle has 32 empty bottles.

Use an array to answer the following questions:

a) How many crates can Lihle fill? How many bottles are left?

b) How many crates does he need to transport all his bottles?

c) How full is the last crate?

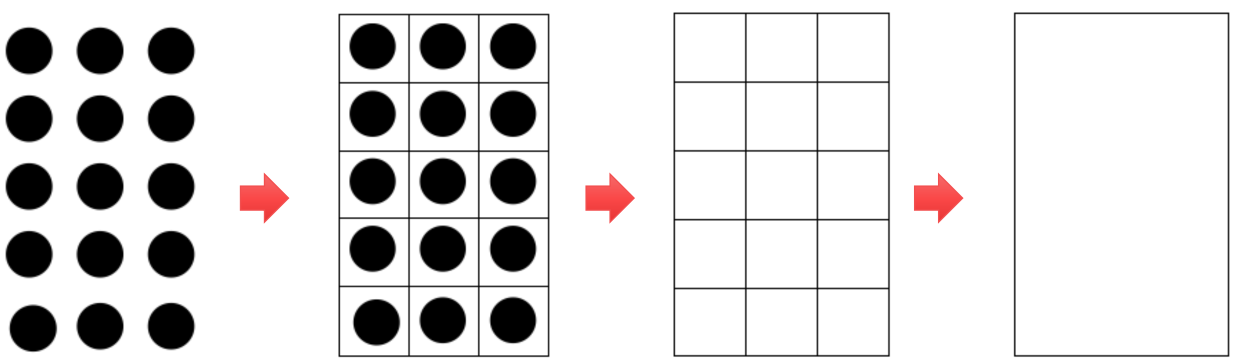
Discussion guide

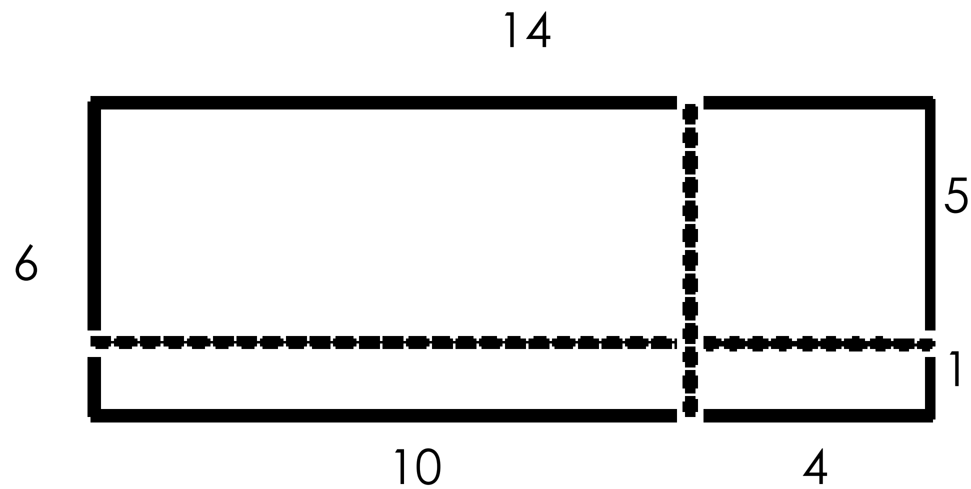
Discuss the diagrams and images that were created:

- Some will work in ones (one bottle at a time).
- Some will work in rows (one row or one column at a time).
- Some will chunk (using a doubling, or a repeated chunk of a known fact).

Look for examples (about 3 or 4 different ways of working) across the group and ask them to REHEARSE and RECORD to bring to the font and explain. Let each person explain what they did. Allow some discussion. If no one uses chunks, then introduce some chunking for this task.

Discuss progression for this representation. It takes time for children to use and refine the array. By the end of grade 3 learners should be able to use the area model for multiplication. This is a very useful model for multiplication in the higher grades.





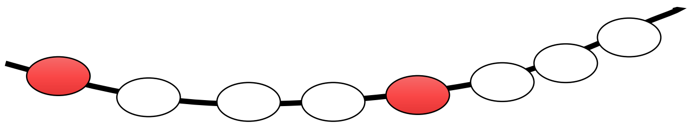
This rectangle showing 14 x 6, is an example of the area of a rectangle model.

The area or a rectangle model can be used to multiplication with bigger numbers. This is an example for 14 across by 6 down (14 x 6). Draw is on the board. Allow the teachers to work with each smaller array first: 10 by and 4 by 5, 10 by 1 and 4 by 1. Add together all the arrays to get the big array. Try this with some other problems they have already worked on (e.g. 5 by 15 for the muffins problem).

T-table as representation

Task 24: Beads

I am making a necklace. This is the pattern I will repeat.



a) If I have 2 red beads, how many white beads do I need?

b) If I have 15 white beads, how many red beads do I need?

Red	White
1	3
2	
3	
4	12
5	

c) Design your own necklace problem.

d) Solve your problem using a T table.

Discussion guide

The T-Table builds on the array representation. Pay attention to the labels at the top (red and white). T-tables give a record of the multiplicative relationship.

Use a T-table to record the muffins problem. There are 5 muffins in each row. There are 75 muffins all together. Make a table with the headings: rows and muffins.

Children take time to come to use and to 'bring to mind' a T-table. This is used more in Grade 3 (but can be introduced in Grade 2). It is easier to completing a T-table than make a T-table from scratch. All time and opportunity for children to choose the headings

Task 25: Frog family goes walking again

Mommy frog and baby frog go for a walk. When mommy frog does one big jump, baby frog does two small hops to keep up.

If mommy frog does 5 jumps, how many hops does baby frog have to do?

Mommy	Baby
1	2
2	4

Daddy frog takes big jumps. When he takes one big jump, baby frog takes five little hops to get to the same place.

a) If daddy frog does 3 jumps, how many hops does baby frog have to do to get to the same place?

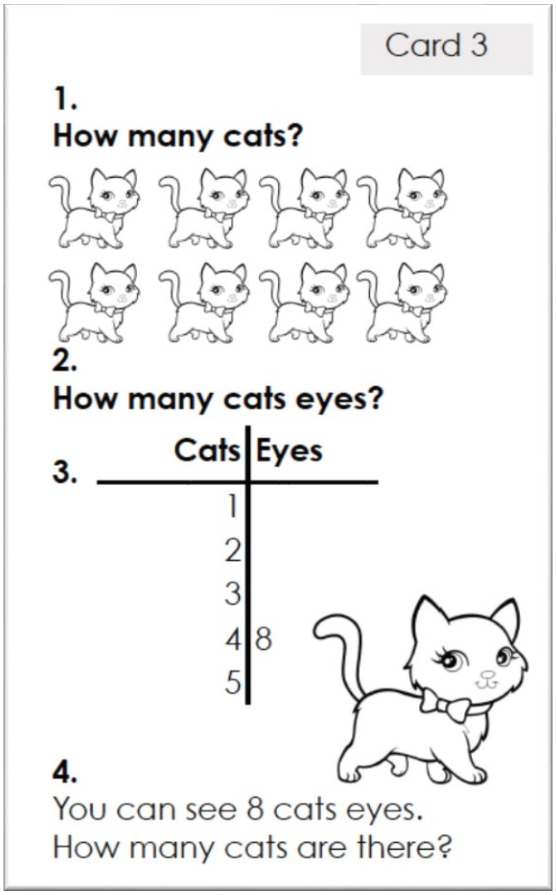
Daddy	Baby
1	5
2	10

Discussion guide

The T-table just the number line turned on its side (rotated through 90º).

It is important to label the headings.

In the resource pack, there are a number of fluency homework cards that require learners to use a T-table. Note that for the T-table the numbers in the left-hand column increase by 1?

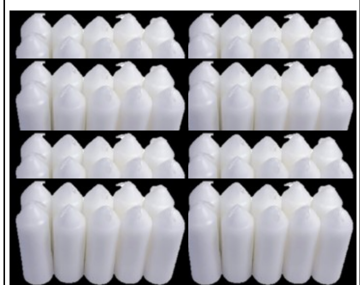


T-tables have all the multiplicative relationship for all the numbers starting at 1. This is an appropriate strategy when the numbers are small but when they become big it takes too long to work out all the numbers in between. For example, if there was a question on Card 3 that read: 'You can see 86 eyes, how many cats are there? Therefore, we introduce more chunking methods (using a clueboard or area of a rectangle model or other formal written methods). This is what is needed in Intermediate Phase.

Task 26: Doubling packs of candles

One pack has 10 candles.

How many candles in each picture?

				
1 pack	2	4	8	10
10 candles	20			

Discussion guide

Clueboards are a special kind of T-table. A clue board is a very important image for reasoning about multiplication and division. It is a tool that helps many learners who struggle to recall their timetables well. To make a clueboard, children only need to know how to double and how to multiply by 10.

The basic clueboard uses the input values 1, 2, 4, 8 and 10; and the concept of doubling. You get bigger inputs by multiplying by ten to give 10; 20; 40; 80 and 100 as input values.

The figure below shows a clueboard for multiplying by 6.

×6	
1	6
2	12
4	24
8	48
10	**60**

There are also examples of fluency cards for T-tables in the resource pack provided. Note that for the T-table the order in the right-hand column is 1, 2, 4, 8, 10, (5)